CHAPTER 8

MATHEMATICAL MODELLING WITH A GRAPHICS CALCULATOR

FIONA GRANT AND JOHN SEARL

The Texas Instruments Calculator Based Laboratory used with graphics calculators, enables students to undertake mathematical modelling through problem solving using classroom generated data. The easy transfer of data to each student's calculator allows self–paced, independent work either individually or in small groups. There are opportunities for discussion between students and teacher and between the students themselves. The activities can facilitate cross–curricular links between mathematics and the sciences. They provide a means for concept development in the students and they demonstrate practical applications of mathematics. This study has evaluated the use of the CBL in a number of Scottish schools. It has shown that the activities are rich in mathematical concepts, that they can be tailored to the mathematical maturity of the students and that they enhance the quality of learning.

INTRODUCTION

Calculators are becoming more sophisticated, more widely available and less costly. Many ambitious claims have been made about the benefits of such tools in mathematics education, but we should not rely on enthusiasm, opinions or wishful thinking. Mathematics is a powerful means of communication. It can be used to solve problems, describe reality and test assertions. Experimentation, observation and reflective criticism will inform us about the potential role of calculators in these aspects of mathematics, so that technology may enhance the teaching and learning environment. In this paper we will describe one of a series of small scale studies involving the use of calculators in Scottish schools.

The Texas Instruments Calculator Based Laboratory (CBL) is a battery powered, portable, data collection device. A variety of probes can be connected to the CBL so that data such as temperature, light intensity, sound levels and distance can be collected. The data is sent to a graphics calculator for analysis. The equipment enables students to undertake mathematical modelling through problem solving using real data.

This study has evaluated the use of the CBL in a number of Scottish secondary schools, during 1995/96. The project involved 12 schools and 530 learners in the Lothian region. The activities were used with whole classes of pupils, aged from 13+ (S2) to 17+ (S6) years, in class sizes rang-

P. GÓMEZ & B. WAITS (EDS.). *ROLES OF CALCULATORS IN THE CLASSROOM*, 71-86

ing from 8 to 30. Classroom observation, record sheets and discussions with the class teachers all provided feedback which assisted in the development and modification of the activities.

The activities involve pupils in collecting and analyzing data using TI–82 graphics calculators. The easy transfer of data to the students' calculators, via the link cable, facilitates self–paced, independent work, either individually or in small groups. The use of an overhead–projector calculator and screen allows whole class viewing and involvement with large groups of pupils.

PUPIL GENERATED DISTANCE–TIME GRAPHS

The ultrasonic motion detector records the distance of an object or person, in the range 0.5m to 6m. Pupils move in front of the sensor to create their own distance–time graphs. The graph is produced on the screen at the same time as the person moves.

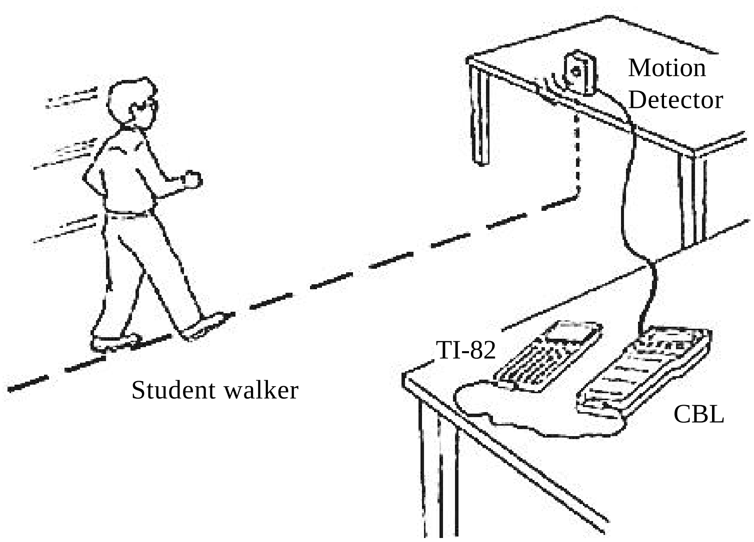

Figure 1.

Pupils describe the graph and relate its shape to their speed and direction of movement. Discussion of different graphs (straight–line, parabolic, discontinuous, etc.) and whether all graphs are possible encourages learners to verbalize their thoughts. Stationary points take on real meaning!

Straight–line graphs can be generated by the calculator for pupils to match by walking the journey. Pupils analyze the graphs for starting and finishing distances, speed and direction.

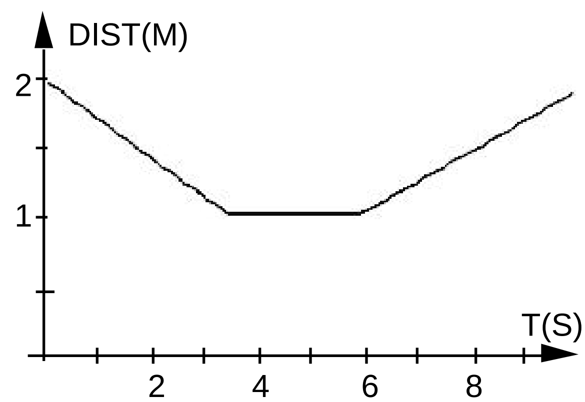

Figure 2.

The activities are relevant in the sense of being personally meaningful. We have found that pupils not only find the activities fun but that they can give pupils of all abilities a better intuitive understanding of distance–time graphs. Immediate feedback and the possibility of trial and improvement encourages discussion and friendly advice from their peers. Pupils are often keen to pose and test out their own questions. For example, what would the graph look like of someone opening the door, entering the room and closing the door? Would there be two separate lines if two people walked in front of the sensor?

We have observed that pupils' previously held beliefs can be challenged, leading to cognitive adaptation through the accommodation of new experience into existing schemas (Skemp,1986). Research has shown that pupils often interpret a distance–time graph as being a picture of the movement in a vertical plane, rather than a relationship between two variables (Kerslake,1981). Brasell (1987) compared the effectiveness of real–time graphing and delayed–time graphing. She found that a delay, even of only 20-30 seconds between the movement and the display, reduced the improvement in the students' understanding of distance–time graphs. ″Real–time graphing allows learners to process information about the event and the graph simultaneously rather than sequentially.″ (Brasell, 1987, p. 386). This puts less of a strain on short–term memory. Our observations support Brasell's view that the dynamic aspect of real–time graphing may motivate learners to focus on the key features of the graph, such as changes in speed or direction.

When the pupils talk about and produce the graphs, the teacher gains some insight into their current levels of understanding. We have observed pupils' concepts developing and even changing through the process of describing, predicting and testing. For example, the possibility of the graphs in Figure 3 has generated interesting discussion.

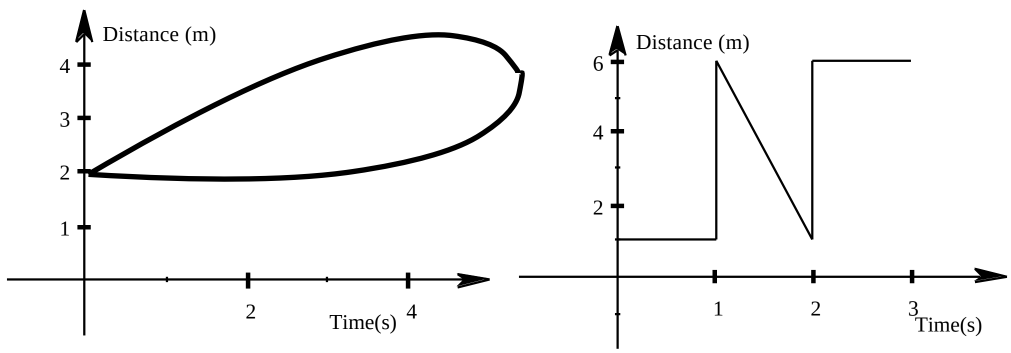

Figure 3.

For (a) pupils have suggested walking in a loop and returning to the starting point. This is an example of pupils believing that the distance–time graph is a picture of the movement. Another suggestion is to have two people walking at the same time. By testing their predictions, pupils find out for themselves that the graph is not possible and then justify their conclusion by comments such as:

it is not possible to travel back in time

the sensor cannot record the distances of two people simultaneously.

In (b), the horizontal lines cause no difficulty but many pupils are convinced they can produce a vertical line. Suggestions include jumping, running very fast or forming a line of people who will move out of the range of the beam in turn. Again, by testing out their ideas, pupils discover for themselves that the faster the speed, the steeper the gradient. Since an object cannot travel a finite distance in zero time, most pupils realize that a vertical line is impossible. Pupils have offered explanations such as

you cannot travel a distance without taking up any time *you can't be in all the places on the verticle (*yes!*) line at the same time"*

Some pupils, however, were resistant to change their strongly held view; offering justification such as

it would be possible if you could travel fast enough

Following verbalization in class, pupils express their ideas in writing, which encourages them to clarify their thoughts. This written record gives the teacher insight into each pupil's thinking which is not always possible in a large group discussion. We have noticed in the discussion about a parabolic graph, for example, many pupils make verbal observations about change of speed but in the written descriptions many pupils still write that you move with constant speed, slowing down only to change direction. It would appear that many pupils see a parabola as two straight lines joined by an arc, rather than a curve with continuously changing gradient.

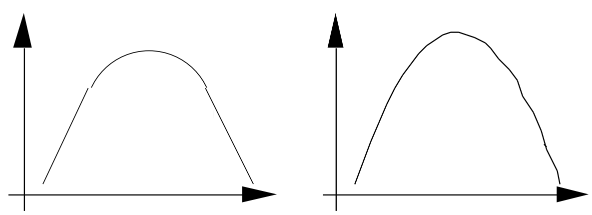

Figure 4.

For pupils who did not usually perform well at written mathematics, participation in the distance–time graph activities gave them an opportunity to succeed mathematically. The activities and tasks that pupils encounter during a mathematics lesson influence their attitude towards the subject. Some teachers commented how surprised they were to observe pupils who generally experienced great difficulty in written tasks, taking part so successfully in these activities. Teachers also mentioned the potential contribution of the activities to the pupils' confidence and motivation.

In the introductory activities to distance–time graphs, the interpretations are mainly qualitative. The graphs can, however, be analyzed more quantitatively in terms of speed and rates of change, leading to calculus concepts. Estimations of speed can be made by finding the gradient of a chord. Pupils observe that this is constant for a straight line and varies for a curve. This leads to a discussion of positive and negative gradients, as well as zero and infinite gradients. Another activity allows comparison between the distance–time and velocity–time graph of a walk in front of the sensor.

Pupils find an approximate value for the area under the velocity–time graph, by summing the areas of rectangles, and relate this to the distance travelled.

Figure 5.

LIGHT INTENSITY

The light probe records the light intensity of a light source at various distances. An ordinary household light bulb or a bicycle lamp both work well.

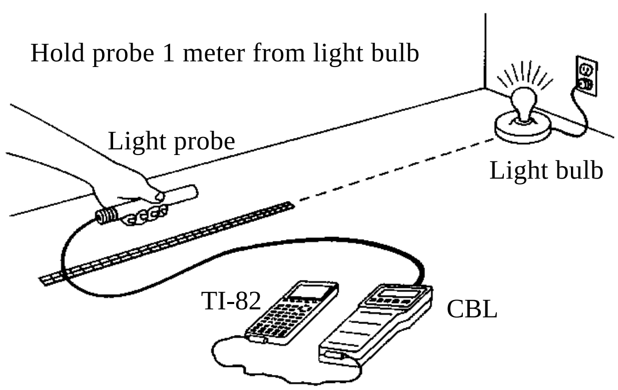

Figure 6.

From a light source, *O*, the light radiates onto the surface of a sphere. The area illuminated is given by $a = 4\pi r^2$. The light intensity, *I*, is the amount of light per unit area, *W* is the strength of the light source and r is the radius of the sphere.

$$I = \frac{W}{A} = \frac{W}{4\pi r^2}$$

Hence, the light intensity is inversely proportional to the square of the distance.

$$I = \frac{k}{r^2}$$

where *k* is the constant of proportionality dependent on the light source.

This practical demonstration of inverse variation is an example where the mathematical model is fairly easy for the students to construct themselves. Pupils find a value for *k* and consider how well the theoretical model fits the experimental data. The method used to find the value of *k* will depend on the mathematical maturity of the students. S4 pupils have used trial and improvement by considering certain data points. Older pupils have used a method of least squares fit.

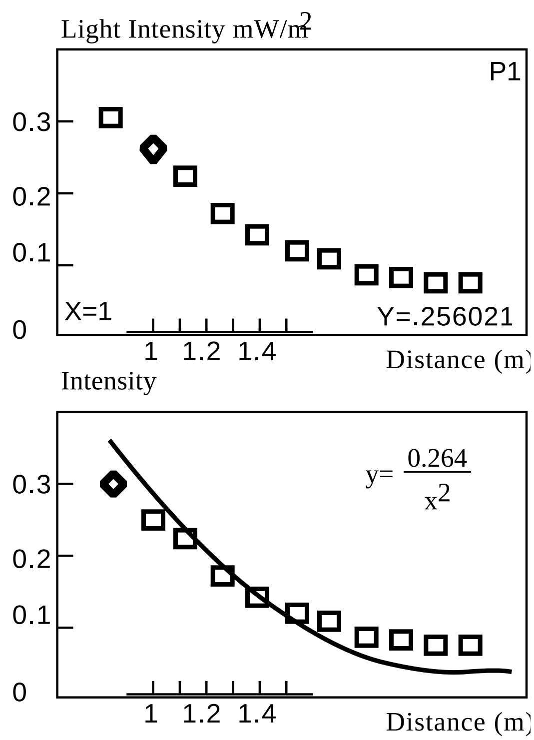

Figure 7.

SOUND WAVES

The microphone records the sound from a vibrating tuning fork, musical instrument or human voice. Pupils can apply their understanding of periodicity, amplitude, phase shift and radian measure to model the sound wave by a sinusoid. They can compare different musical instruments and see the phenomena of beats and harmonics.

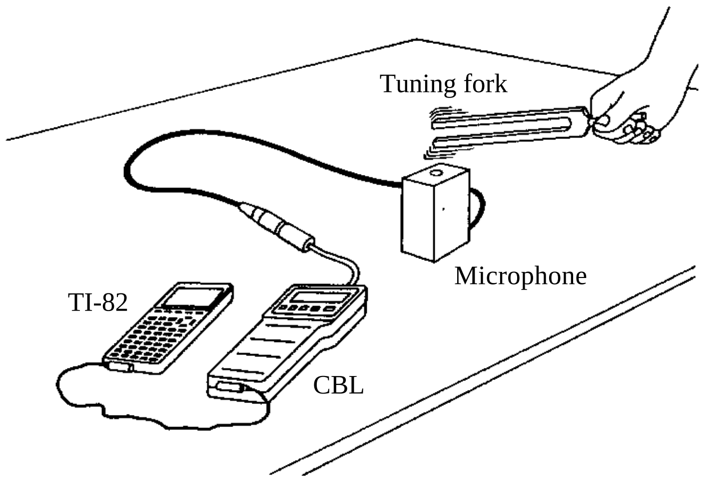

Figure 8.

We have noticed that pupils enjoy seeing the sound wave of a note they have produced themselves and appear to be motivated to model their "own" data. Since the coefficients involved are not small integers, unlike most text book questions, pupils cannot easily "guess" their values. In the process of fitting the graph to the data, pupils develop an intuitive understanding of how the coefficients affect the graph.

BOUNCING BALL

A ball bounces under the motion sensor and a distance time graph is displayed.

This activity is very rich in mathematics, although satisfactory data collection is sometimes problematic! Pupils can analyze various aspects of the data such as individual bounces, bounce heights or bounce times.

Pupils use the TRACE button to read the coordinates of the bounce heights from the graph. Alternatively, by using the data lists, more

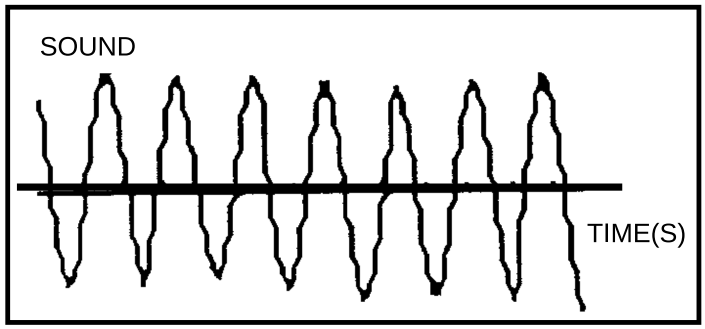

Figure 9.

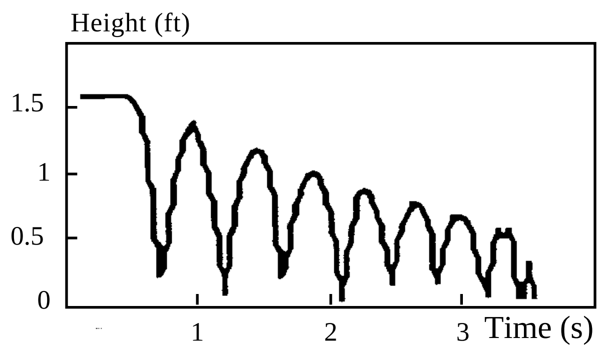

Figure 10.

advanced pupils calculate the maximum height of each bounce by considering three points around the maximum and fitting a quadratic equation through those points. By calculating the ratios of successive bounce heights, pupils can observe a relationship between the heights and verbalize this example of a geometric sequence. Comments from pupils in S4 who had not yet encountered the term "geometric sequence", but who had found a common ratio of approximately 0.87, included:

The ball rebounds to 87% of its previous height

The next height is 0.87 of the one before

If you dropped the ball from 1m it should bounce back up to about 87cm

This understanding of the physical situation enabled them to construct an exponential equation. Pupils plot the individual bounce heights and use the information about the starting height and the bounce ratio to fit an exponential curve through the data points.

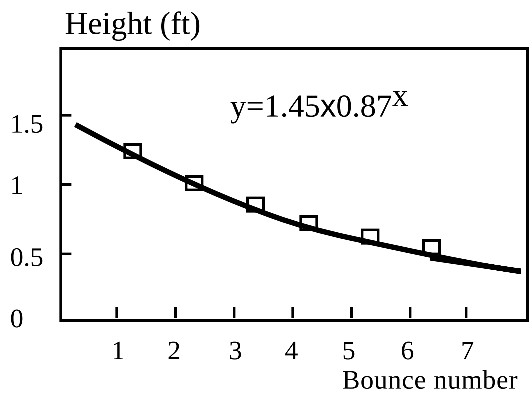

Figure 11.

The times between each bounce also form a geometric sequence which can be analyzed in a similar way to the above modelling procedure. This idea can be developed by summing a geometric series to show that, although there are theoretically an infinite number of bounces, the ball comes to rest in a finite amount of time.

Pupils plot the data from one bounce. This forms a parabolic section in time. Using the vertex form of the quadratic equation, $y = a(x-p)^2 + q$, pupils use the coordinates of the turning point to find an equation which fits the experimental data. They estimate and improve on a value for *a*. We have observed many students who start with a positive value of *a*, discover it must be negative and are then able to give a reason why. The immediate feedback from the calculator screen appears to motivate the pupils to persevere without relying on the teacher's help. The model is superimposed on the original data.

WATER COOLING

The temperature probe is placed into hot water. The probe is left in the water to measure the temperature as the liquid cools. For a shorter data collection time, after being placed into hot water, the probe can be placed into water at room temperature or held in the air to cool.

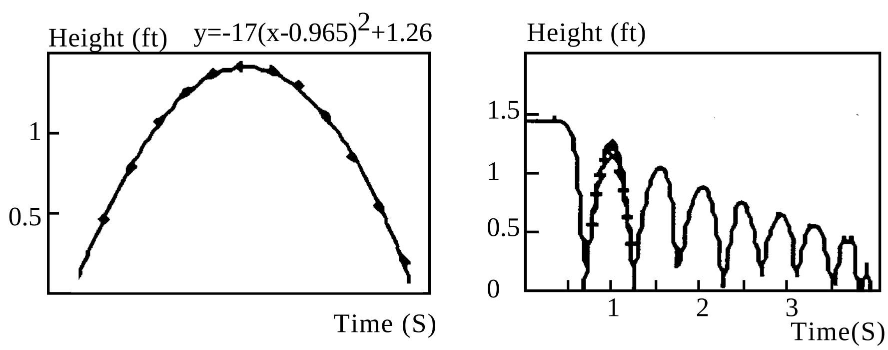

Figure 12.

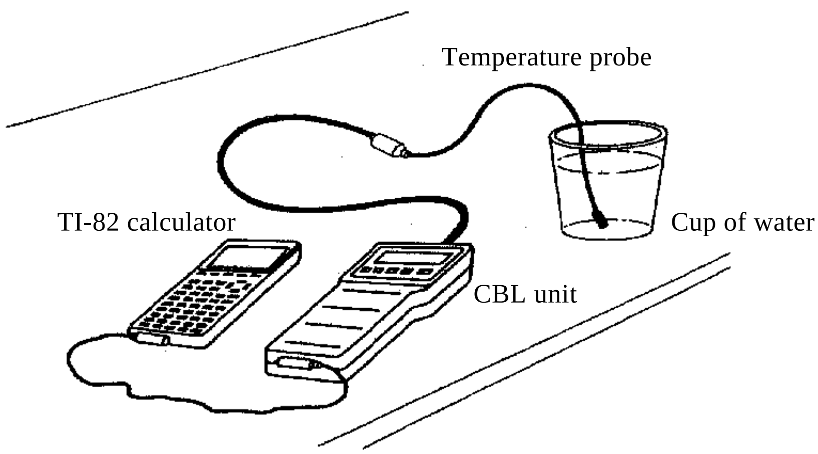

Figure 13.

The pupils model the experimental data in this example of exponential decay. Pupils can compare the insulating effects of different containers and relate their results to Newton's law of cooling. The probe can also be placed into a container of iced water.

This activity has not been as widely used as the others mentioned in this paper. This is because exponential and logarithmic functions tend to be taught towards the end of the Higher course and, therefore, it was not perceived to be as relevant to the pupils as the other activities, at the time of the project.

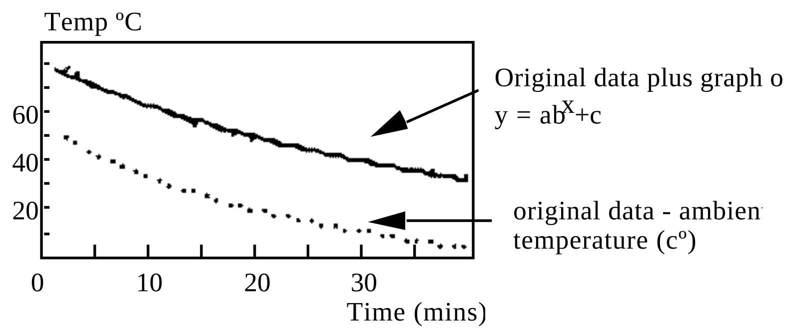

Figure 14.

DISCUSSION

In all of these activities, the management of the learning environment is the teacher's responsibility. The teacher decides when to use the activity, either as an introduction to a topic, as an application, or as consolidation of text book "theory". The teacher also decides whether the data collection will be performed in small groups or by the whole class. We have found that it is feasible for a group of pupils to do the experiment in front of the whole class, using an OHP calculator and screen so that everyone can see the experimental data. This facilitates class discussion before the data is transferred to the pupils' calculators and keeps all pupils involved. Alternatively, each group of pupils could collect their own set of data. For example, with one class of 8 pupils, each pair performed the sound experiment either by playing a note on a musical instrument or by humming. When the data has personal meaning for the pupils they have a genuine mathematical problem to solve, not a pseudo–problem imposed by the teacher or the textbook. This provides motivation and intrinsic reward for the learners and it helps to make mathematics fun! It also helps to demonstrate that mathematics is not a purely bookbound subject, but that it has relevance to the real world.

The teacher also decides how much initial support and direction to give to the pupils. This could include the provision of record sheets or guidance in using the facilities of the graphics calculator, or the task could be left completely open for the learners to explore. This allows for differentiation within the class. It is important that such differentiation is "determined largely by the abilities of the pupils and their achievements within the activity itself and is not predetermined by the teacher." (DES,1985, p.27)

Most of the pupils in S5 or S6 had already used a graphics calculator. Usually this was the Casio fx–7000G rather than a Texas Instruments calculator, since this was the model most commonly found in Lothian schools at the time of the project. One S4 class already had some experience of the

TI–82, but pupils in S4 usually had no experience of any graphics calculator. Pupils with experience of a different model quickly adapted to the TI–82. Pupils with no experience of a graphics calculator needed time to become familiar with some of the facilities such as using the TRACE key to pick out points on a graph or how to plot a graph of two data lists. Teachers also need time to become familiar with the calculator, in order to feel confident when they use them in the classroom and to be able to deal with problems such as graphs "disappearing" when a StatPlot is accidentally turned off! Even if the data cannot be retrieved it is quite simple to transfer it from another calculator.

Unfamiliarity with the calculator sometimes led to the concentration needed to "press the right button" obscuring the mathematics. A few pupils showed a lack of confidence in using the calculators. This was evidenced by their timidity and by comments such as "It always goes wrong when I touch it" when something unexpected appeared on the screen. Conversely, some pupils (mainly boys) were so keen to explore the calculator facilities that they pressed keys randomly, without pausing to think about the mathematics of the task.

The activities provided opportunities for discussion between the pupils and teacher and between the pupils themselves (Cockcroft, 1982). Pupils asked their own questions and pursued aspects which interested them. Pupils expressed their own ideas, formulated their own hypotheses and tested them against experimental evidence. They tried to fit the experimental results to the theoretical model and to account for any differences. In a constructivist view of learning, knowledge is constructed by the individual and not passively received from an external source, often the teacher (von Glaserfeld,1990). While it is the individual who does the constructing, social interaction with others (the teacher and peers) and their prior knowledge and experience are also influential in the learner's concept development process. “students construal of mathematics included making sense of what teachers told them just as it included making sense of their own discoveries” (Jaworski, 1994, p. 85)

The teacher provides activities which the pupils enter into at their own level of understanding. Through interaction with the pupils the teacher gains a sense of the levels of understanding. Growth in understanding is a dynamic process of reorganization (Pirie and Kieran,1992). Rather than planning and rigidly applying a prepared teaching sequence, the teacher “must be constantly reappraising the learning taking place within the classroom environment as it evolves.” (Pirie and Kieran, 1992). Activities which involve data collection are unlikely to produce identical results when repeated. Learners contribute their own ideas to the experiments and to the modelling process. This “non–replicable” aspect of a practical activity provides a freshness which may be missing from a familiar, often repeated exposition. The teacher approaches the activity “as if” anew and gains fresh insight into it. The teacher may challenge beliefs which do not fit with the generally held view and try to guide the learner to a deeper understanding. The activities, therefore, also have a diagnostic function, both in the oral discussion and in the pupils' written records. “Because understanding is an internal state of mind which has to be achieved individually by each pupil, it cannot be observed directly by the teacher […] A much better indication of the depth of understanding which exists can be obtained in the course of discussion, by means of appropriate practical work or through more general problem–solving activities.” (Cockcroft, para 232)

Many of the CBL activities are appropriate for use in many different subject areas in addition to mathematics, such as biological and physical sciences, agriculture, medicine, music and statistics. Cross–curricular links are facilitated even though the approach may be different in the various subjects. Coursework projects could be undertaken, either individually or in small groups. Flexible study timetables are possible with the use of this personal, detached technology.

An electricity supply is not essential, unless the OHP is used. This means that data can be collected outside the school building or in places without electricity, such as in rural areas of developing countries. Computer printouts of the screen graphs, data lists or programs can be obtained via a graph link cable. Learners can keep a hard copy of their experimental data and save it on disk for further analysis. Access to the computer, however, does not need to be immediate. This would be useful where a mathematics department or a school did not possess a computer but where one was available elsewhere. This is relevant to rural schools in developing countries, where the nearest computer may be in the capital city or nearest town.

Practical activities have been widely recommended (Cockcroft, 1982, DES,1985) but they rarely occur in the secondary mathematics classroom. “There is a tendency to minimize the importance of […] practical work at the later primary stage and at the secondary stage. Without sufficient practical experience the pupils are unable to relate abstract mathematical concepts to any form of reality. All pupils benefit from appropriate practical work whatever their age or ability.” (DES,1985, p. 39)

This lack of practical work is partly due to the time needed both to set up apparatus and for the pupils to do the activity. Experimental data is not usually neat and tidy like a textbook question. Errors and sensible degrees of accuracy need to be taken into account. Teachers are often uncertain about the benefits of practical activities. They no longer feel in control of what the pupils will learn and prefer not to risk “wasting” teaching time. Syllabi are perceived to be overcrowded with content. Teachers, especially of upper secondary school classes, feel under pressure from assessment procedures and, in many countries, from external examinations. Many teachers do not feel confident about using new technology in the classroom. Practical activities and modelling can, therefore, be considered a distraction from the perceived main task, which is to ensure the students pass the examination. Teacher development, through pre–service and in–service courses, is important. Teachers need time to challenge and examine their beliefs about mathematics. Personal convictions affect classroom behavior (Ahmed,1987). The Cockcroft report (1982, para 243) recommended seven elements which should be included in mathematics lessons:

- exposition by the teacher
- discussion between teacher and pupils and between pupils themselves
- appropriate practical work
- consolidation and practice of fundamental skills and routines
- problem–solving, including the application of mathematics to everyday situations
- investigational work

The activities described here contribute to providing a learning environment appropriate for all pupils. The activities assist in concept development by providing pupils with opportunities to discuss, challenge misconceptions, negotiate ideas and build their own understanding in a relaxed atmosphere. The focus is not on algorithmic procedures but on graphical interpretation and developing mathematical models using realistic contexts.

The study has shown that the activities are rich in mathematical concepts. They can be tailored to the maturity of the learners and, when used appropriately, they enhance the quality of learning.

REFERENCES

Ahmed, A. (1987). *Better Mathematics: A Curriculum Development Study*. London: HMSO.

Brasell, H. (1987). The Effect of Real–Time Laboratory Graphing on Learning Graphic Representations of Distance and Velocity. *Journal of Research in Science Teaching* 24, 4, pp 385–395.

Brueningsen, C. et al (1994). *Real–World Math with the CBL™ System*. Dallas: Texas Instruments.

Cockcroft, W.H. (1982). *Mathematics Counts*. London: HMSO.

DES (1985). *Mathematics from 5 to 16 Curriculum Matters 3*.London: HMSO.

Jaworski, B. (1994). *Investigating Mathematics Teaching*. London: Falmer Press.

Kerslake, D. in Hart K.M. (1981). (Ed.). *Children's Understanding of Mathematics 11-16*. London: John Murray..

Pirie, S. and Kieran, T. (1992). Creating Constructivist Environments and Constructing Creative Mathematics. *Educational Studies in Mathematics*, 23, pp505-528.

Skemp, R. (1986). *The Psychology of Learning Mathematics*. Harmondsworth: Penguin

von Glasersfeld, E. (1990). An Exposition of Constructivism: Why some like it radical. In Davis R.B., Maher, C.A. and Noddings, N. (Eds.). *Constructivist Views on the Learning and Teaching of Mathematics Journal for Research in Mathematics Education Monograph Number 4.* Reston: NCTM.

Fiona Grant
fiona@maths.ed.ac.uk

John Searl
searl@maths.ed.ac.uk

Edinburgh Centre for Mathematical Education
University of Edinburgh
JCMB
Kings Buildings
Mayfield Road
Edinburgh
EH9 3JZ
Scotland