CHAPTER 17

# THE CHANGE OF THE METHOD IN MATHEMATICS EDUCATION WITH TI-82 IN JAPAN

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*Up to now Japanese mathematics education has been focused on formality and abstract notions, with a lot of time spent doing calculations. This has to change, and the introduction of the graphing calculator might help in this process. With graphing calculators, mathematics teaching and learning can become more interesting. In this paper this is shown through examples in which the calculator is used in precalculus, differential calculus and problem solving. Now many students can see and touch the mathematical knowledge using the calculator, and enjoy their time in the mathematical lessons.*

## JAPANESE SCHOOL SYSTEM

Japanese think that education is very important in making a good society. So we had schools *Terakoya* at Edo (200 years ago), and 100 years ago we built up many elementary schools in all areas. Parents have thought that children must go to school everyday. The percentage of school attendance is high: Kindergarten 84%, Elementary School 100%, Junior high school 100%, High School 98%, and University 47%. We think that the Japanese economic growth is the consequence of school education.

In Japan, Elementary School and Junior High School are compulsory. Some pupils take the entrance examination to the Private Junior high school. If they do not go to the Private Junior High School, they can enter the Public School with no examination. But many parents want their children to go into a good private school. Since calculators have not been allowed on entrance examinations, they are not used either in the mathematics classroom in either elementary and junior high school.

The Japanese government decides the curriculum for all levels through what is called the *course of study.* All teachers teach the same content to their pupils and use the same textbooks which have to be approved by the Ministry of Education. Japanese teachers have a good idea of how to teach mathematics without calculators. However, calculator use has become compulsory, and now we must use the calculator at 5th year mathematics lesson at the Elementary School. This change may make a new mathematics education in Japan.

P. GÓMEZ & B. WAITS (EDS.). *ROLES OF CALCULATORS IN THE CLASSROOM*, 197-206

## JAPANESE MATHEMATICS EDUCATION

The main characteristic of Japanese mathematical education is training in calculation. The students study methods of calculation without the calculator. They use the abacus very often. The Japanese students spend most of their time in calculation drills without calculators. Why do they spend so much time on this in the mathematics lessons? Teachers and parents think that the power of mathematics is in calculating ability. So they think that "to do mathematics" is "to do calculation". Teachers evaluate students on their ability to calculate. Students use paper and pencil for mathematical calculations in every lesson and they are fast at doing these calculations. This has had two implications. One is that the Japanese performance is very good on the IEA, the International Association For The Evaluation Of Educational Achievement tests. The other is that Japanese students do not develop a process for thinking in mathematics.

## WHY DO NOT WE USE THE CALCULATOR AT MATHEMATICS EDUCATION IN JAPAN?

Manual (with abacus) and mental calculation are very important in Japanese mathematics education. We think that to calculate with numbers is to understand the mathematical concept. The ability to calculate is the ability to do mathematics. Studying mathematics is understanding the calculation. Therefore, many Japanese parents and teachers are anxious about the declining achievement in calculations. This point is the greatest deterrent to calculator use. They do not want their children to use calculators on school lessons. The second trouble is that calculators are not allowed on the entrance examinations. However, Japanese rely on calculators in the daily life, and pupils use the calculators all the time after school. They even use the calculator to do their homework with the help of their parents.

The following are examples of manual calculations at the end of Elementary School.

1) $35 + 7 \times 2 - 12 \div 3 - 8 =$

$80 \div (25 - 15) - 2 \times 3 =$

$(12 \div (18 \div (4 + 5) + 2)) =$

2) $0.25 \times 0.04 =$

$0.072 \times 0.016 =$

3) $\frac{1}{5} + \frac{2}{5} \times \frac{1}{4} =$

$\frac{3}{4} \div \frac{9}{10} - \frac{1}{3} =$

The following are examples of the use of manual calculators (abacus):

1) $(2.65 \times 102.03 - 2.65 \times 2.03) =$

$20 \times 3.14 \times 5 =$

2) $2.21 + 6.5 \div \frac{1}{2} =$

$4.5 \times \frac{2}{3} + 5.37 =$

3) $0.3 + \frac{1}{4} + 0.05 \times (4.03 - 1.15) \div \frac{2}{5} =$

4) $\frac{13}{25} + 0.05 \times (4.03 - 1.15) \div \frac{2}{5} =$

In the mathematics class, we want to teach the ability to do creative thinking. It is thought that if students drill and can do mental calculations well, they will apply them to improving the efficiency of manual calculation with an abacus. The following are considered to be good uses of mental calculations in improving the efficiency of manual calculations at the elementary level.

1) $2.65 \times 102.03 - 2.65 \times 2.03$

$= 2.65 \times (102.03 - 2.03)$ common number

$= 2.65 \times 100$

$= 256$

2) $20 \times 3.14 \times 5$

$= 3.14 \times 20 \times 5$ commutative law

$= 3.14 \times 100$

$= 314$

3) $97 \times 103$

$= (100 - 3) \times (100 + 3)$ applied factorization

$= 10000 - 9$

$= 9991$

1) $97 + 96 + 103 + 98 + 101 + 103$

$= 100 \times 6 + (-3 - 4 + 3 - 2 + 1 + 3)$

$= 600 + (-2)$

$= 598$

The reason for not using calculators is that students are not usually presented problems where a calculator would be a useful tool. Some textbooks explicitly prohibit the use of calculators for solving some of the problems, since, if used, the problems would become too easy and senseless. We are therefore missing the advantages of calculators as tools that enable students to approach interesting mathematical problems.

## THE IEA AND THE LACK OF CALCULATORS IN JAPANESE MATHEMATICAL EDUCATION

The performance of Japanese students in the IEA has been very good. The second international mathematics study (1980) was a comprehensive survey of the teaching and learning of mathematics in the schools of twenty countries. The first study was done in 1964 in thirteen countries. The Japanese data in these tests show two results. First, Japanese students show a high level of performance on the calculation aspects of the test, but their performance is low on the reasoning aspects of it, when compared to the performance of students from other countries.

| **Easy**<br>**Addition of common fraction** | | **Difficult**<br>**Square root** | |
|---|---|---|---|
| | $\frac{2}{5} + \frac{3}{8}$ is equal to | | What is the square root of 12*75 |
| A | $\frac{5}{13}$ | A | 6.25 |
| B | $\frac{5}{40}$ | *B | 30 |
| C | $\frac{6}{40}$ | C | 87 |

| **Easy**<br>**Addition of common fraction** | | **Difficult**<br>**Square root** | |
|---|---|---|---|
| D | $\frac{16}{15}$ | D | 625 |
| *E | $\frac{31}{40}$ | E | 900 |
| Percentage of correct answers from Japanese | | Percentage of correct answers from Japanese | |
| 85% | | 7% | |

The first problem is done many times at school. However, the second is not, and this might explain the differences in correct answers from Japanese students. They have not solved this kind of problem and expect the teacher to teach it to them.

## THE ROLE OF THE GRAPHING CALCULATOR IN THE FUTURE IN JAPAN

Since we think in Japan that mathematics education is drilling on calculation, we do not see a need to use the calculator. But now, *the course of study* for 5th grade allows pupils to use the calculator and that may help us change the teaching style in the mathematics classroom. Pupils will be able to become creative, they will be able to "see" the mathematics, and, perhaps, they will enjoy doing mathematics. With calculators we cannot insist on just looking at whether the answer is correct or not. We will have to take into account the way the answer is produced.

## USING CALCULATORS IN MY LESSONS

Mathematical lessons are very important for the science student. The student studies very hard to learn calculating methods without the use of computers or calculators. So the student who can calculate very well is valued highly. This kind of assessment is not the best since Japanese students do not have a good sense of mathematical thinking. With the new technology the way mathematics is taught will change. We will go from calculating the expressions to studying the concept.

I have used the graphing calculator in my lessons. It was very delightful to teach these lessons to my students. They gave an outlet to their latent energy to learn mathematical concepts. They can now understand the con-

cept of mathematics. The new technology allows them to see the mathematical objects and to approach and solve new types of problems.

## CHANGE THE TEACHING OF MATHEMATICS WITH THE TI-82

I think it is very important to use the graphing calculator in the teaching and learning of mathematics. The following are some examples of calculator use.

### The parametric function and its motion as it is graphed

Draw the graph of the parametric function

$$x_1 = \cos t$$
$$y_1 = \sin t$$

Japanese students can draw the graph very quickly. They can eliminate the parameter t, and get the function $x^2 + y^2 = 1$. This graph is the circle with center in the origin and radius equals to 1. This problem is very easy for Japanese students.

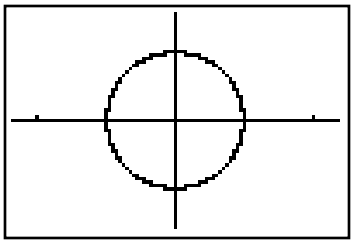
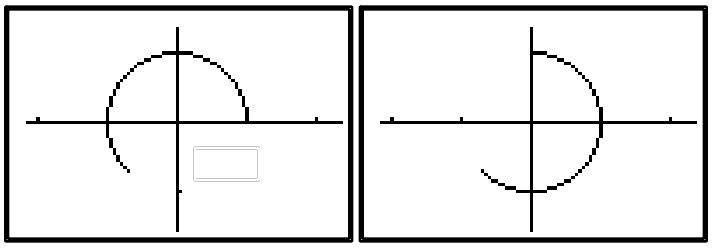


*Figure 1. Unit circle*

However, if the problem is changed to finding the differences between the parametric functions

$$\begin{matrix} x_1 = \cos t \\ y_1 = \sin t \end{matrix} \quad \textit{and} \quad \begin{matrix} x_2 = \sin t \\ y_2 = \cos t \end{matrix},$$

then Japanese students can not understand the difference between the two functions. They get the same unit circle. With the help of the graphing calculator, we can see the difference: One turns to the right, the other takes a left turn.



$x_1 = \cos t$ $\quad$ $x_2 = \sin t$

$y_1 = \sin t$ $\quad$ $y_2 = \cos t$

*Figure 2. Two different circles*

## Seeing the functions: Fourier series

We used the graphing calculator in my lessons on Fourier series. The Fourier series is one branch of mathematics that is very important for our science students. In Japanese mathematics courses, we teach the representation of the Fourier series of given functions. If it is possible to integrate the function, we calculate the period function to the Fourier series with the Euler-Fourier formula. This calculation is easy for Japanese students. But they do not understand the meaning of this calculation. We have the main elements in the representation of calculating with the Euler-Fourier's formula. And if we can calculate the given function to the Fourier series, then we finish this lesson. This calculation is the main purpose of the mathematics lesson. Next we teach the convergence of the Fourier series. The convergence is more difficult for students. They do not understand it and resist further development in mathematics. But now, with this new technology, they can see the form of given functions, and show interest in the representations of the Fourier series.

We calculate the Fourier series of the function

$$y = \begin{cases} -1 \text{ if } x \in [-\pi, 0] \\ 1 \text{ if } x \in [0, \pi] \end{cases}$$

Students can calculate to Fourier series, but they do not know what it means. The expression of the Fourier series is

$$y = \frac{4}{\pi}\left( \sin x + \frac{\sin 3x}{3} + \frac{\sin 5x}{5} + \ldots \right)$$

We use the graphing calculator to draw the graph of this Fourier series function.

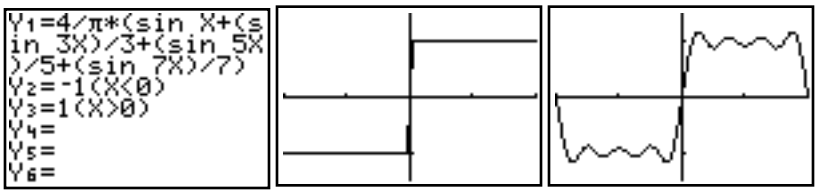



*Figure 3. Graph of a function and its Fourier series*

## Problem solving learning is exploring: maximum and minimum of functions

The graphing calculator allows the student to explore, to see the mathematical objects and, therefore, to enjoy the mathematics lesson. The technology empowers the student for doing mathematics. We show the example of finding the locus of the maximum or minimum of functions. In this example we are surprised by the very wide axes. We do not find this kind of problem in the textbooks.

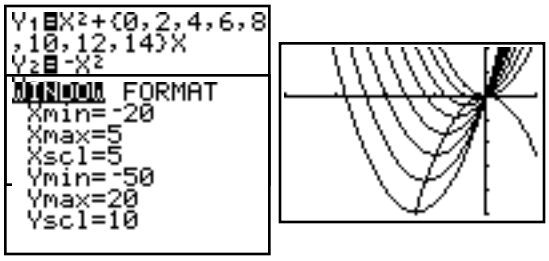



*Figure 4. The locus of apex of the parabola $y = x^2 + ax$ is $y = -x^2$*

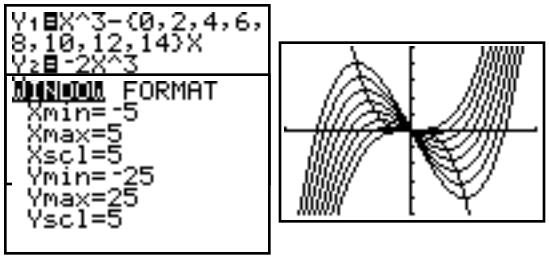



*Figure 5. The locus of the maximum or minimum of $y = x^3 - ax$ ($a \geq 0$) is $y = -2x^3$*

## Differential calculus

Without the calculator, students spend most of their time drawing the graphs of the functions. With the calculator, they can focus on the charac-

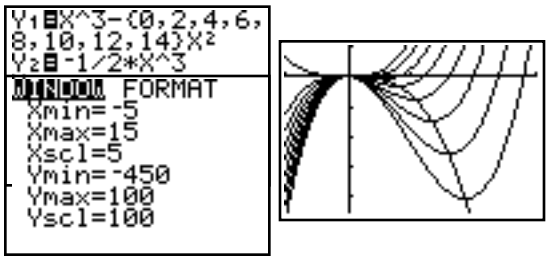



*Figure 6. The locus of the minimum of $y = x^3 + ax$ ($a \geq 0$) is $y=-0.5\ x^3$*

teristics of the function. For example, when we see the graph of $y = x^3 - 3x$, we can find the special point, the intercept with the x-axis and the y-axis, and the extreme points. And we want to calculate these

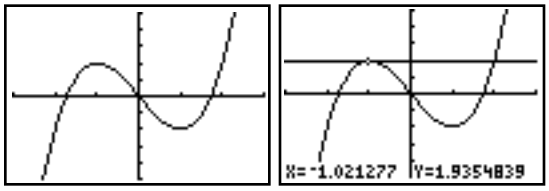



*Figure 7. Graph of $y = x^3 - 3x$*

points. In this example, the horizontal line is drawn near the maximum point (-1, 2). It is not exact but the value is very near. We can easily find the point with differential calculus. It is very difficult to find out the point of inflection (0,0). Seeing this graph suggests the existence of the point.

## SEEING AND EXPLORING: UNDERSTAND AND ENJOY MATHEMATICS

We had these mathematics lessons using the graphing calculator. Seeing reinforces the understanding of abstract concepts. Before the calculator, we could not see the mathematical theorems. Students were convinced that they could not understand the mathematical theorems. They took a passive and conservative attitude toward learning and could not approach new problems. But now we have some enjoyment in lessons with calculators. The new technology, TI-82 and TI-92, are good machines for learning mathematics. We are using the graphing calculator everyday in the class.

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